$\boxed{16-51}$

$$V_B = \omega r = 8(0.15) = 1.2\,m/s$$

$$\vec{V}_B = 1.2\cos 60\,\vec{i} + 1.2\sin 60\,\vec{j}$$

$$\vec{V}_A = V_A\vec{i} \quad , \vec{\omega}_{AB} = -\omega_{AB}\vec{k}$$

$$\vec{r}_{A/B} = 0.5\cos 60\,\vec{i} + 0.5\sin 60\,\vec{j}$$

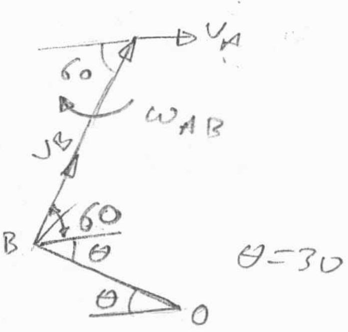
$$\vec{V}_A = \vec{V}_B + \vec{\omega}_{AB} \times \vec{r}_{A/B}$$

$$V_A\vec{i} = 1.2\cos 60\,\vec{i} + 1.2\sin 60\,\vec{j} - 0.5\,\omega_{AB}\vec{k} \times (\cos 60\,\vec{i} + \sin 60\,\vec{j})$$

$$V_A i = (1.2\cos 60 + 0.5\,\omega_{AB}\sin 60)\,i + (1.2\sin 60 - 0.5\,\omega_{AB}\cos 60)\,j$$

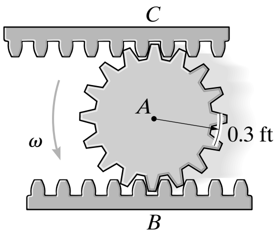
j: $$\omega_{AB} = \frac{1.2\sin 60}{0.5\cos 60} = 4.157$$

i: $$V_A = 1.2\cos 60 + 0.5(4.157)\sin 60 = 2.4\ m/s$$



$V_B \perp BO$

16-53 Pinion gear A rolls on the gear racks B and C. If B is moving to the right at 8 ft/s and C is moving to the left at 4 ft/s, determine the angular velocity of the pinion gear and the velocity of its center A.



$$\mathbf{v}_C = \mathbf{v}_B + \mathbf{v}_{C/B}$$

$$(\overset{+}{\rightarrow}) \qquad -4 = 8 - 0.6(\omega)$$

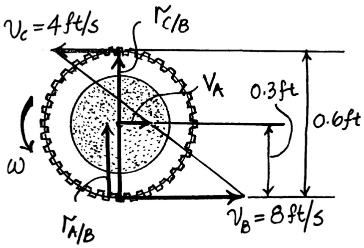
$$\omega = 20 \text{ rad/s}$$ **Ans.**

$$\mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$(\overset{+}{\rightarrow}) \qquad v_A = 8 - 20(0.3)$$

$$v_A = 2 \text{ ft/s} \rightarrow$$ **Ans.**

Also,



$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B}$$

$$-4\mathbf{i} = 8\mathbf{i} + (\omega\mathbf{k}) \times (0.6\mathbf{j})$$

$$-4 = 8 - 0.6\omega$$

$$\omega = 20 \text{ rad/s}$$ **Ans.**

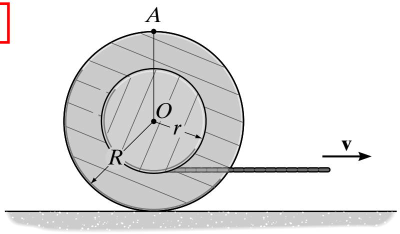
$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{A/B}$$

$$v_A\,\mathbf{i} = 8\mathbf{i} + 20\mathbf{k} \times (0.3\mathbf{j})$$

$$v_A = 2 \text{ ft/s} \rightarrow$$ **Ans.**

16-55 Determine the velocity of point A on the outer rim of the spool at the instant shown when the cable is pulled to the right with a velocity of $\mathbf{v}$. The spool rolls without slipping.

extra similar to 16-55



Kinematic Diagram: Since the spool rolls without slipping, the velocity of the contact point P is zero. The kinematic diagram of the spool is shown in Fig. *a*.

General Plane Motion: Applying the relative velocity equation and referring to Fig. *a*,

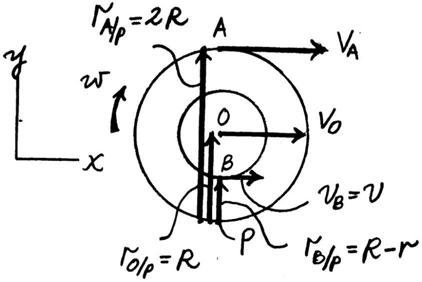
$$\mathbf{v}_B = \mathbf{v}_P + \boldsymbol{\omega} \times \mathbf{r}_{B/D}$$

$$v\mathbf{i} = \mathbf{0} + (-\omega\mathbf{k}) \times \left[(R - r)\mathbf{j}\right]$$

$$v\mathbf{i} = \omega(R - r)\mathbf{i}$$

Equating the $\mathbf{i}$ components, yields

$$v = \omega(R - r) \qquad \omega = \frac{v}{R - r}$$



(a)

Using this result,

$$\mathbf{v}_A = \mathbf{v}_P + \boldsymbol{\omega} \times \mathbf{r}_{A/P}$$

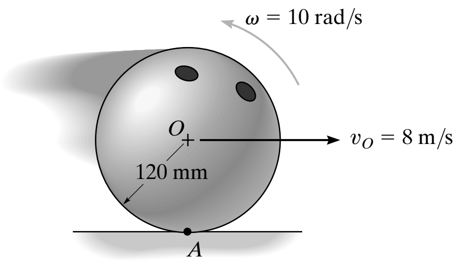
$$= \mathbf{0} + \left(-\frac{v}{R - r}\mathbf{k}\right) \times 2R\mathbf{j}$$

$$= \left[\left(\frac{2R}{R - r}\right)v\right]\mathbf{i}$$

Thus,

$$v_A = \left(\frac{2R}{R - r}\right)v \rightarrow$$ **Ans.**

16-56 A bowling ball is cast on the "alley" with a backspin of $\omega = 10$ rad/s while its center O has a forward velocity of $v_O = 8$ m/s. Determine the velocity of the contact point A in contact with the alley.

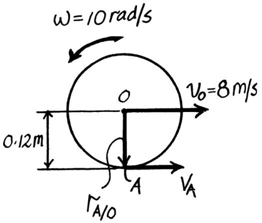


$$\mathbf{v}_A = \mathbf{v}_O + \mathbf{v}_{A/O}$$

$$\left(\overset{+}{\rightarrow}\right) \qquad v_A = 8 + 10(0.12)$$

$$v_A = 9.20 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$

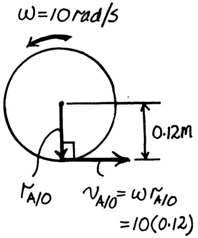
Also,



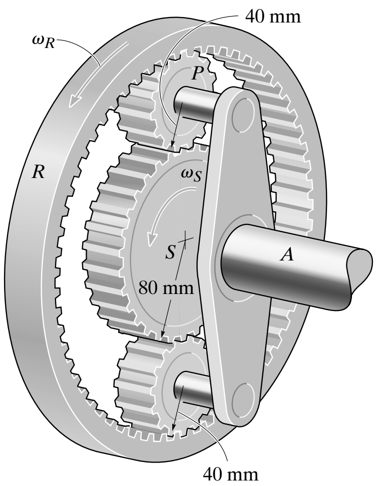
$$\mathbf{v}_A = \mathbf{v}_O + \boldsymbol{\omega} \times \mathbf{r}_{A/O}$$

$$v_A\mathbf{i} = 8\mathbf{i} + (10\mathbf{k}) \times (-0.12\mathbf{j})$$

$$\left(\overset{+}{\rightarrow}\right) \qquad v_A = 9.20 \text{ m/s} \rightarrow \qquad \textbf{Ans.}$$



16-63 The planetary gear system is used in an automatic transmission for an automobile. By locking or releasing certain gears, it has the advantage of operating the car at different speeds. Consider the case where the ring gear R is held fixed, $\omega_R = 0$, and the sun gear S is rotating at $\omega_S = 5$ rad/s. Determine the angular velocity of each of the planet gears P and shaft A.



$$v_A = 5(80) = 400 \text{ mm/s} \leftarrow$$

$$v_B = 0$$

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

$$0 = -400\mathbf{i} + (\omega_p\,\mathbf{k}) \times (80\mathbf{j})$$

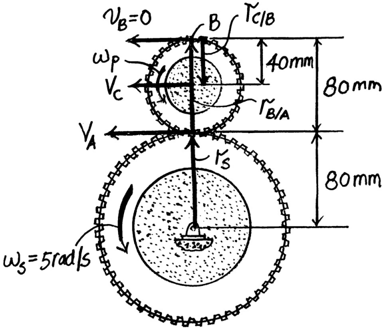
$$0 = -400\mathbf{i} - 80\omega_p\,\mathbf{i}$$

$$\omega_P = -5 \text{ rad/s} = 5 \text{ rad/s}$$ **Ans.**

$$\mathbf{v}_C = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r}_{C/B}$$

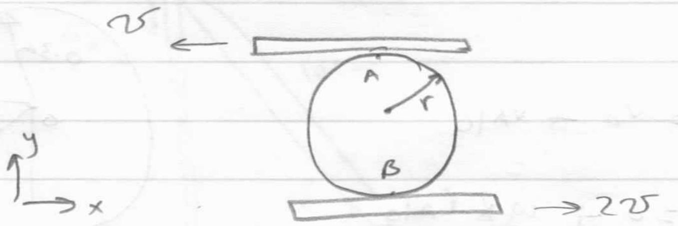
$$\mathbf{v}_C = 0 + (-5\mathbf{k}) \times (-40\mathbf{j}) = -200\mathbf{i}$$

$$\omega_A = \frac{200}{120} = 1.67 \text{ rad/s}$$ **Ans.**



H.W. #5

16-91



$\vec{V}_B = 2v\,i$

$\vec{V}_A = -v\,i$

$\vec{V}_A = \vec{V}_B + \vec{V}_{A|B}$

$-vi = 2vi + \omega \times r_{A|B}$

$-vi = 2v\,i + \omega_d k \times 2r\,j$

$-vi = 2v\,i - 2r\omega_d\,i$

$-3v\,i = -2r\,\omega_d\,i$

$\Rightarrow \omega_d = +\frac{3v}{2r} = 1.5\,\frac{v}{r}$ ((counter clockwise)

16-87

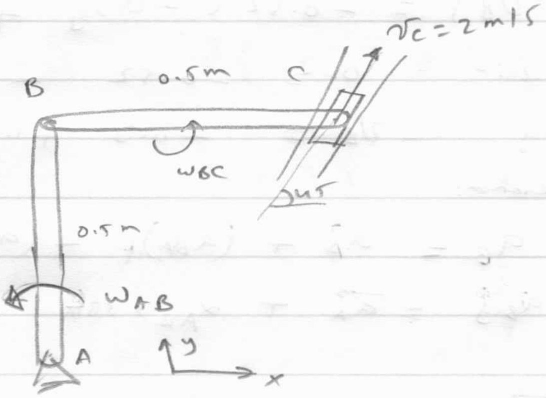
For AB

$\vec{V}_B = \vec{V}_A + \vec{V}_{B|A}$

$= 0 + \omega_{AB} \times r_{B|A}$

$= \omega_{AB} k \times 0.5 j$

$= 0.5\,\omega_{AB}\,i$



For BC :-

$\vec{V}_C = \vec{V}_B + \vec{V}_{C|B}$

$\vec{V}_C = \vec{V}_B + \omega_{BC} \times r_{C|B}$

$2(\cos 45\,i + \sin 45\,j) = 0.5\,\omega_{AB}\,i + \omega_{BC}\,\cancel{k} \times 0.5\,i$

$2\cos 45\,i + 2\sin 45\,j = 0.5\,\omega_{AB}\,i + 0.5\,\omega_{BC}\,j$

For j :-

$2\sin 45 = 0.5\,\omega_{BC} \Rightarrow \omega_{BC} = 2.83$ rad/s

For i : $2\cos 45 = 0.5 * \omega_{AB} \Rightarrow \omega_{AB} = 2.83$ rad/s

$V_B = 0.5 \times 2.83\,i$

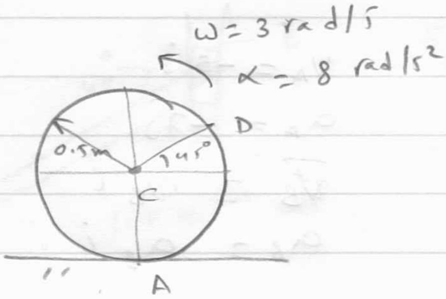
$= 1.41\,i$

(1.41 m/s $\Rightarrow$)

16-114

No slipping at A:

$v_A = 0$



$$\vec{V_C} = \vec{V_A} + \overrightarrow{V_{C|A}}$$

$$\vec{V_c} = 0 + \omega \times r_{C|A}$$

$$= 3k \times 0.5j$$

$$\vec{V_c} = -1.5\,i$$

C moves in rectlinear motion

$$a_c = \alpha r = 8 * 0.5 = 4 \text{ m/s}^2 \leftarrow$$

$$\vec{a_c} = -4i$$

$$\vec{a_D} = \vec{a_c} + (a_{D|C})_t + (a_{D|C})_n$$

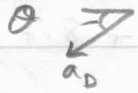
$$= -4i + \alpha \times r_{D|C} - \omega^2 r_{D|C}$$

$$= -4i + 8k \times (0.5\cos 45\, i + 0.5 \sin 45\, j) - 3^2 (0.5\cos 45\, i + 0.5\sin$$

$$= -4i + 2.82j - 2.82i - 3.18i - 3.18j$$

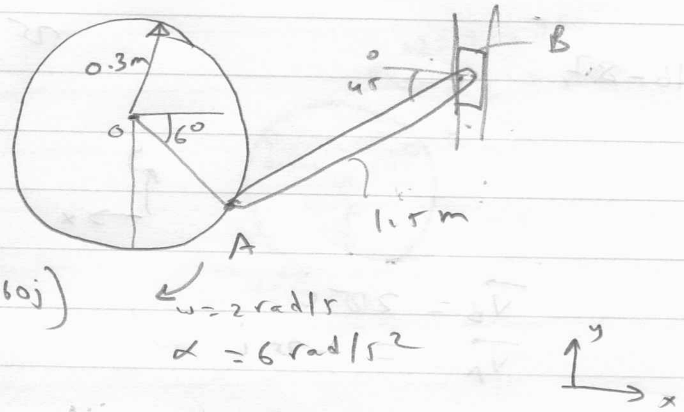
$$= -10i - 0.36j$$

$$|\vec{a_D}| = \sqrt{10^2 + 0.36^2} = 10.006 \text{ m/s}^2$$



$$\theta = \tan^{-1} \frac{0.36}{10} = 2.06^\circ$$

16-123



$v_A = v_0 + v_{A/O}$

$= 0 + \omega_d \times r_{A/O}$

$= -2k \times (0.3 \cos 60 i - 0.3 \sin 60 j)$

$= -0.3 j - 0.52 i$

$r_{B/A} = 1.5 \cos 45 i + 1.5 \sin 45 j$

$= 1.06 i + 1.06 j$

$r_{A/O} = 0.3 \cos 60 i - 0.3 \sin 60 j$

$= 0.15 i - 0.26 j$

$\vec{V_B} = \vec{V_A} + \overrightarrow{V_{B/A}}$

$= -0.52 i - 0.3 j + \omega_{AB} \times r_{B/A}$

$v_B j = -0.52 i - 0.3 j + \omega_{AB} k \times (1.06 i + 1.06 j)$

$v_B j = -0.52 i - 0.3 j + 1.06 \omega_{AB} j - 1.06 \omega_{AB} i$

For i :- $0 = -0.52 - 1.06 \omega_{AB} \Rightarrow \omega_{AB} = -0.49$ rad/s

For j: $v_B = -0.3 + 1.06 (-0.49) \Rightarrow v_B = -0.82$ m/s

acceleration:

$a_B = \vec{a_A} + (a_{BA})_t + (a_{B/A})_n$

$\vec{a_B} j = \vec{a_A} + \alpha_{AB} \times r_{B/A} - \omega_{AB}^2 r_{B/A}$

$\vec{a}_A = \vec{a_o} + (a_{A/O})_t + (a_{A/O})_n$

$= 0 + \alpha_d \times r_{A/O} - \omega_d^2 r_{A/O}$

$= -6k \times (0.15 i - 0.26 j) - 2^2 (0.15 i - 0.26 j)$

$= -0.9 j - 1.56 i - 0.6 i + 1.04 j$

$= -2.16 i + 0.14 j$

$\Rightarrow a_B j = -2.16 i + 0.14 j + \alpha_{AB} k \times (1.06 i + 1.06 j) - (0.49)^2 (1.06 i + 1.06 j)$

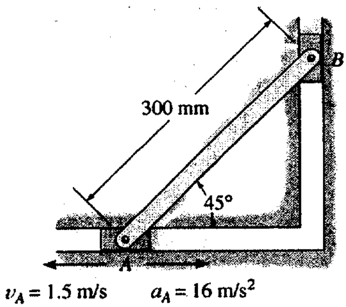
$a_B j = -2.16 i + 0.14 j + 1.06 \alpha_{AB} j - 1.06 \alpha_{AB} i - 0.255 i - 0.255 j$

For i :-

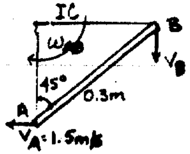
$0 = -2.16 - 1.06 \alpha_{AB} - 0.255 \Rightarrow \alpha_{AB} = -2.28$ rad/s^2

For j :- $a_B = +0.14 + 1.06 \times (-2.28) - 0.255 = -2.53$ m/s^2

***16-108.** At a given instant, the slider block A has the velocity and deceleration shown. Determine the acceleration of block B and the angular acceleration of the link at this instant.



$$\omega_{AB} = \frac{v_B}{r_{A/IC}} = \frac{1.5}{0.3\cos 45^\circ} = 7.07 \text{ rad/s}$$

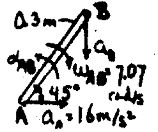


$$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} - \omega^2 \mathbf{r}_{B/A}$$

$$-a_B\,\mathbf{j} = 16\mathbf{i} + (\alpha\mathbf{k}) \times (0.3\cos 45^\circ\mathbf{i} + 0.3\sin 45^\circ\mathbf{j}) - (7.07)^2(0.3\cos 45^\circ\mathbf{i} + 0.3\sin 45^\circ\mathbf{j})$$

$(\overset{+}{\rightarrow})$ $\quad 0 = 16 - \alpha(0.3)\sin 45^\circ - (7.07)^2(0.3)\cos 45^\circ$

$(+\downarrow)$ $\quad a_B = 0 - \alpha(0.3)\cos 45^\circ + (7.07)^2(0.3)\sin 45^\circ$

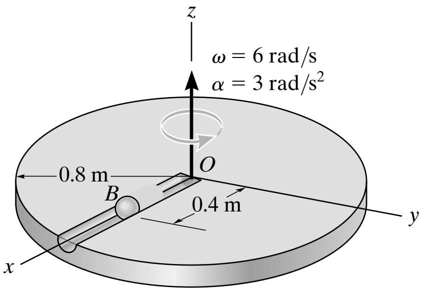


Solving:

$\alpha_{AB} = 25.4$ rad/s^2 ↻ **Ans**

$a_B = 5.21$ m/s^2 ↓ **Ans**

16-131 At the instant shown, ball B is rolling along the slot in the disk with a velocity of 600 mm/s and an acceleration of 150 mm/s^2, both measured relative to the disk and directed away from O. If at the same instant the disk has the angular velocity and angular acceleration shown, determine the velocity and acceleration of the ball at this instant.



Kinematic Equations:

$$\mathbf{v}_B = \mathbf{v}_O + \Omega \times \mathbf{r}_{B/O} + (\mathbf{v}_{B/O})_{xyz} \quad \textbf{(1)}$$

$$\mathbf{a}_B = \mathbf{a}_O + \dot{\Omega} \times \mathbf{r}_{B/O} + \Omega \times (\Omega \times \mathbf{r}_{B/O}) + 2\Omega \times (\mathbf{v}_{B/O})_{xyz} + (\mathbf{a}_{B/O})_{xyz} \quad \textbf{(2)}$$

$\mathbf{v}_O = \mathbf{0}$

$\mathbf{a}_O = \mathbf{0}$

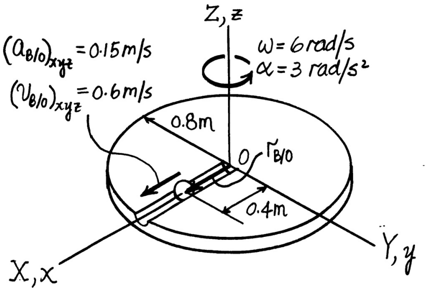
$\Omega = \{6\mathbf{k}\}$ rad/s

$\Omega = \{3\mathbf{k}\}$ rad/s^2

$\mathbf{r}_{B/O} = \{0.4\,\mathbf{i}\,\}$ m

$(\mathbf{v}_{B/O})_{xyz} = \{0.6\mathbf{i}\}$ m/s

$(\mathbf{a}_{B/O})_{xyz} = \{0.15\mathbf{i}\}$ m/s^2



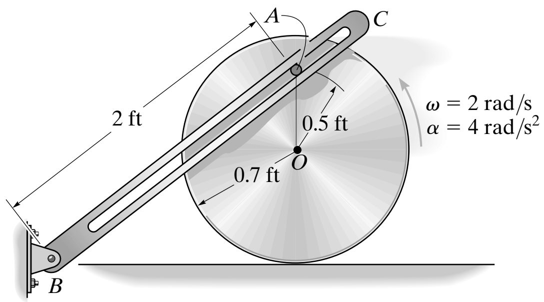
Substitute the data into Eqs.(1) and (2) yields:

$$\mathbf{v}_B = \mathbf{0} + (6\mathbf{k}) \times (0.4\mathbf{i}) + (0.6\mathbf{i}) = \{0.6\mathbf{i} + 2.4\mathbf{j}\} \text{ m/s} \quad \textbf{Ans.}$$

$$\mathbf{a}_B = \mathbf{0} + (3\mathbf{k}) \times (0.4\mathbf{i}) + (6\mathbf{k}) \times [(6\mathbf{k}) \times (0.4\mathbf{i})\,] + 2\,(6\mathbf{k}) \times (0.6\mathbf{i}) + (0.15\mathbf{i})$$

$$= \{-14.2\mathbf{i} + 8.40\mathbf{j}\} \text{ m/s}^2 \quad \textbf{Ans.}$$

16-140 The disk rolls without slipping and at a given instant has the angular motion shown. Determine the angular velocity and angular acceleration of the slotted link *BC* at this instant. The peg at *A* is fixed to the disk.

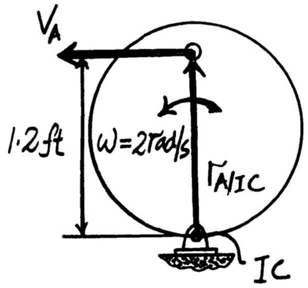


$\mathbf{v}_A = -(1.2)(2)\mathbf{i} = -2.4\mathbf{i}$ ft/s

$\mathbf{a}_A = \mathbf{a}_O + \alpha \times \mathbf{r}_{A/O} - \omega^2\mathbf{r}_{A/O}$

$\mathbf{a}_A = -4(0.7)\mathbf{i} + (4\mathbf{k}) \times (0.5\mathbf{j}) - (2)^2(0.5\mathbf{j})$

$\mathbf{a}_A = -4.8\,\mathbf{i} - 2\mathbf{j}$



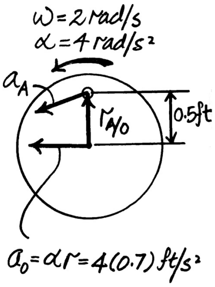
$\mathbf{v}_A = \mathbf{v}_B + \Omega \times \mathbf{r}_{A/B} + (\mathbf{v}_{A/B})_{xyz}$

$-2.4\mathbf{i} = \mathbf{0} + (\omega_{BC}\mathbf{k}) \times (1.6\mathbf{i} + 1.2\mathbf{j}) + v_{A/B}\left(\frac{4}{5}\right)\mathbf{i} + v_{A/B}\left(\frac{3}{5}\right)\mathbf{j}$

$-2.4\mathbf{i} = 1.6\,\omega_{BC}\,\mathbf{j} - 1.2\,\omega_{BC}\,\mathbf{i} + 0.8v_{A/B}\mathbf{i} + 0.6v_{A/B}\,\mathbf{j}$

$-2.4 = -1.2\,\omega_{BC} + 0.8\,v_{A/B}$

$0 = 1.6\omega_{BC} + 0.6v_{A/B}$



Solving,

$\omega_{BC} = 0.720$ rad/s ↺ **Ans.**

$v_{A/B} = -1.92$ ft/s

$\mathbf{a}_A = \mathbf{a}_B + \Omega \times \mathbf{r}_{A/B} + \Omega \times (\Omega \times \mathbf{r}_{A/B}) + 2\Omega \times (\mathbf{v}_{A/B})_{xyz} + (\mathbf{a}_{A/B})_{xyz}$

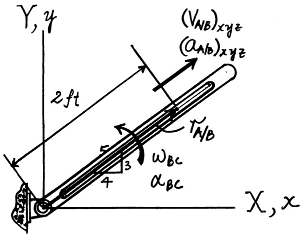
$-4.8\mathbf{i} - 2\mathbf{j} = \mathbf{0} + (\alpha_{BC}\,\mathbf{k}) \times (1.6\mathbf{i} + 1.2\mathbf{j}) + (0.72\mathbf{k}) \times (0.72\mathbf{k} \times (1.6\mathbf{i} + 1.2\mathbf{j}))$

$+2(0.72\mathbf{k}) \times \left[-(0.8)(1.92)\mathbf{i} - 0.6(1.92)\mathbf{j}\right] + 0.8\,a_{B/A}\mathbf{i} + 0.6a_{B/A}\,\mathbf{j}$

$-4.8\mathbf{i} - 2\mathbf{j} = 1.6\alpha_{BC}\mathbf{j} - 1.2\alpha_{BC}\,\mathbf{i} - 0.8294\mathbf{i} - 0.6221\mathbf{j} - 2.2118\mathbf{j} + 1.6589\mathbf{i} + 0.8a_{B/A}\,\mathbf{i} + 0.6a_{B/A}\,\mathbf{j}$

$-4.8 = -1.2\alpha_{BC} - 0.8294 + 1.6589 + 0.8a_{B/A}$

$-2 = 1.6\alpha_{BC} - 0.6221 - 2.2118 + 0.6a_{B/A}$



Solving,

$\alpha_{BC} = 2.02$ rad/s^2 ↺ **Ans.**

$a_{B/A} = -4.00$ ft/s^2